Flipping a Penny

Exploring Randomness and Reviewing Nomenclature

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- Flipping coins, rolling dice, and shuffling cards are usually thought of as random.

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- Now how I might begin the class ....

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- Repeat four times.
- for (i in seq(4)) {
  print(sample(c(0,1), size = 10, replace = TRUE))
  }
  [1] 0 0 0 1 1 0 1 1 1 0
  [1] 0 1 0 1 1 0 1 1 0 1
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- For example,

case	heads	run_length
0 0 0 1 1 0 1 1 1 0	5	3
0 1 0 1 1 0 1 1 0 1	6	2
0 0 1 1 0 0 1 1 1 1	6	4
1 0 1 1 0 0 0 0 0 0	3	6

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- Record the values of your variables in the class spreadsheets "human.csv" and "penny.csv" with column headers "head" and "run_length".

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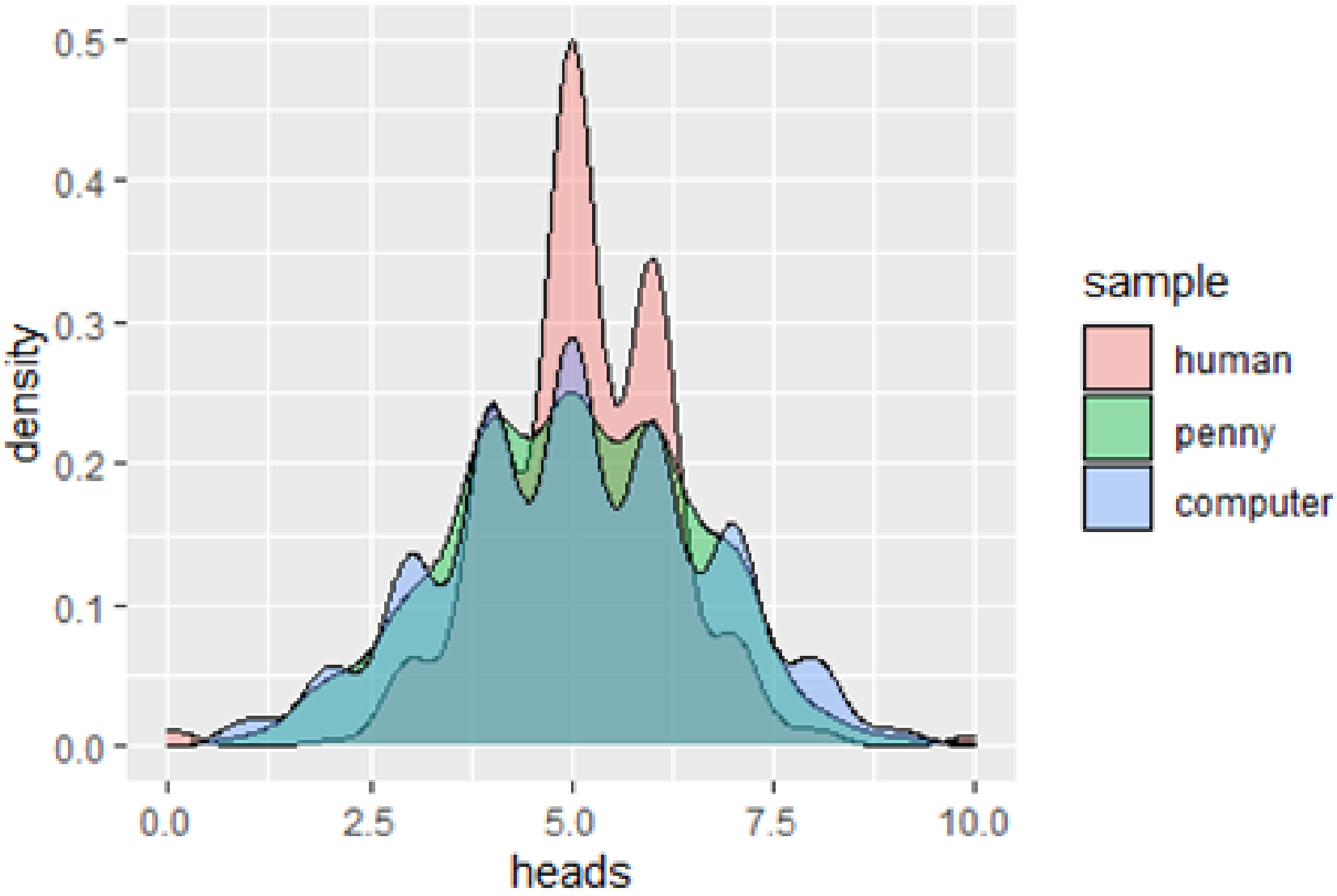
- computer = tibble(heads = c(), run_length = c())
  for (i in seq(1000)) {
  case = sample(c(0,1), size = 10, replace = TRUE)
  one_row = tibble(
  heads = sum(case),
  run_length = max(rle(case)$lengths))
  computer = bind_rows(computer, one_row)
  }
  write_csv(computer, "computer.csv")

Number of Heads Comparison



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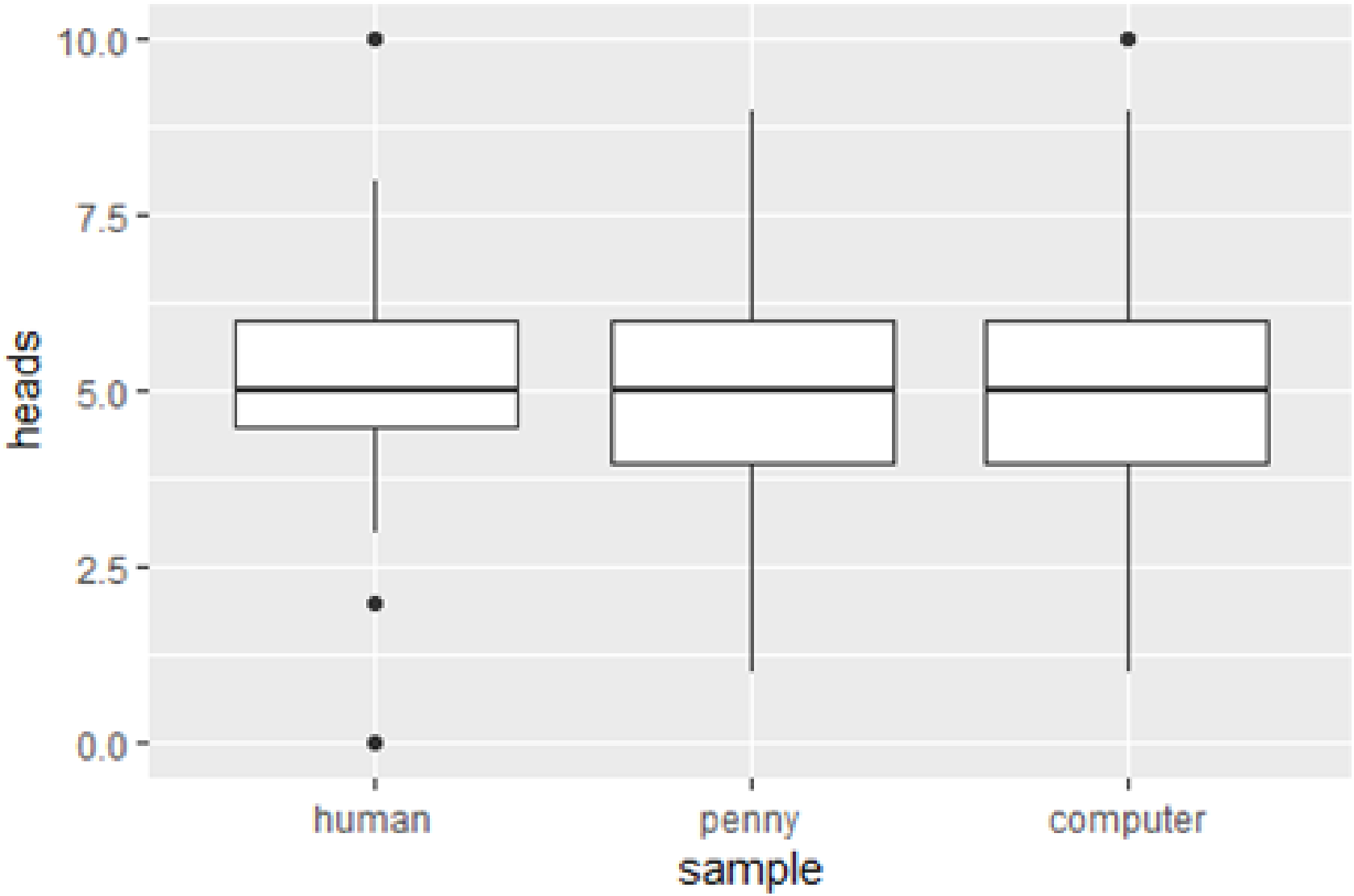
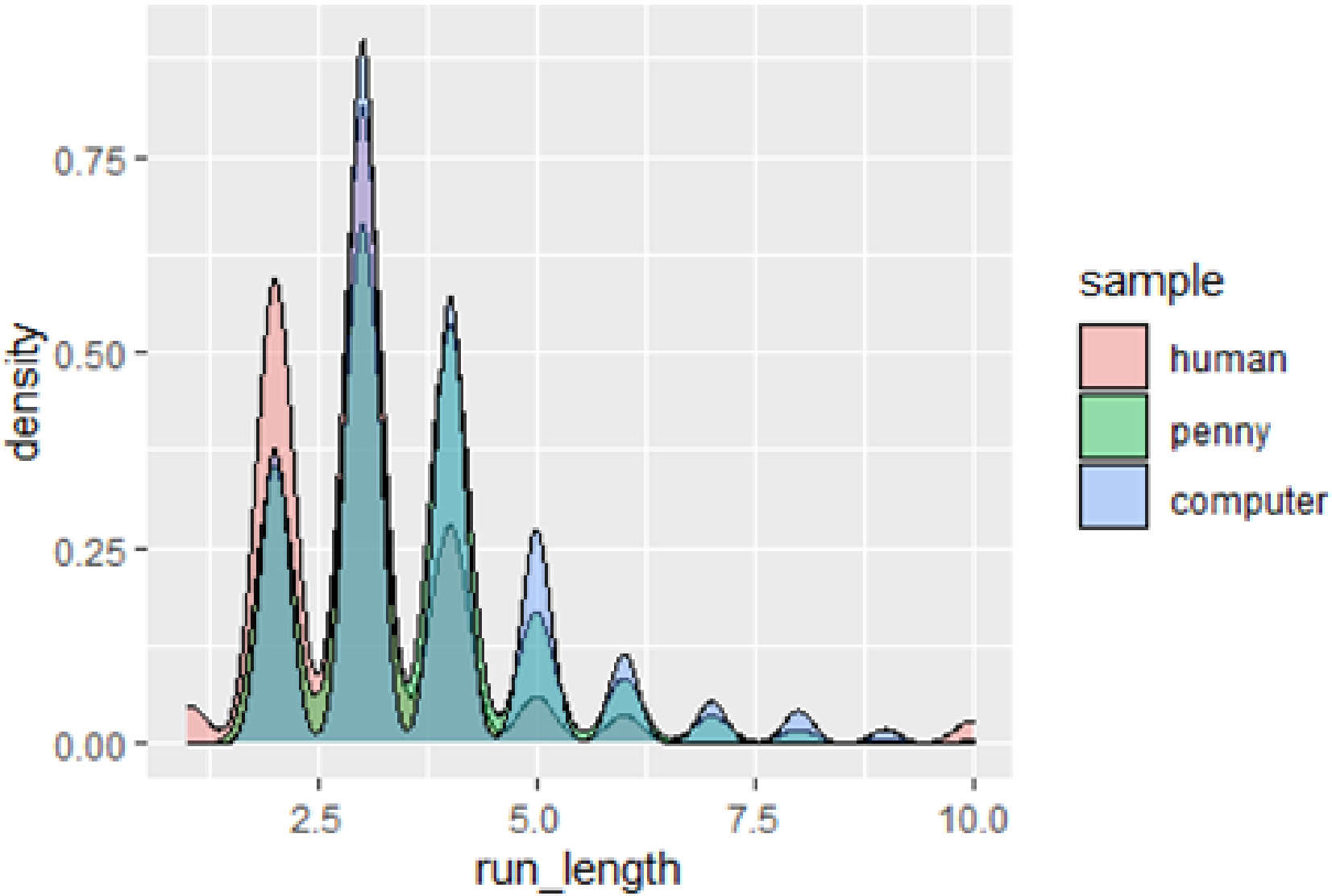


Length of Longest Run Comparison



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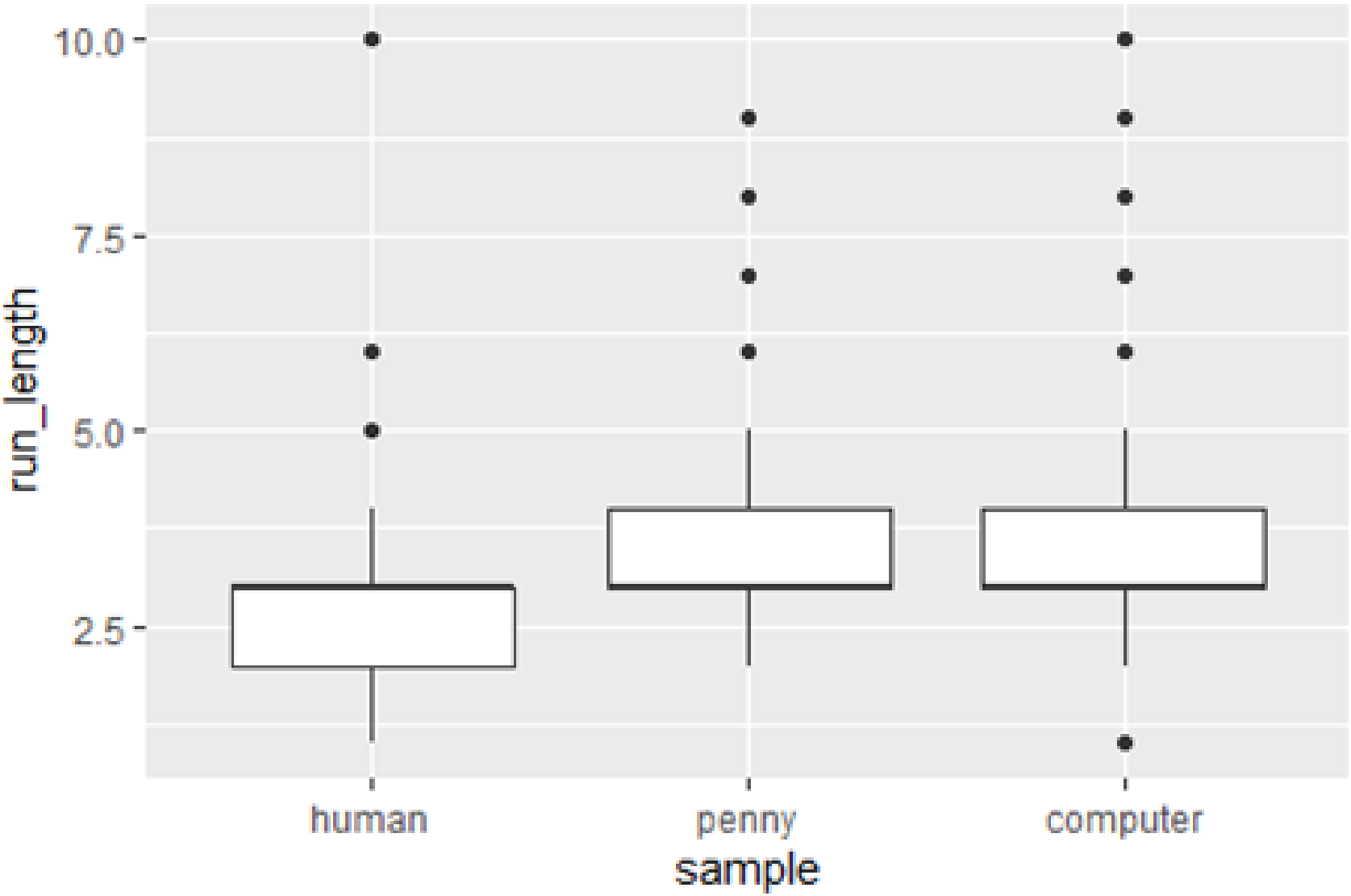

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- Rather than the computer constructing comparison visualizations, students can construct the human simulation and actual data histograms by hand on a piece of graph paper that is projected for everyone to see.
- A fun followup activity is to have students "flip" a penny by first resting the penny on its side on a table and then jostling the table. After 100 or so flips have happened and the percentage of heads has been determined, ask students whether this is a fair way to flip a penny.