My Favorite Statistics Activity: Transformations of Random Variables

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Overview

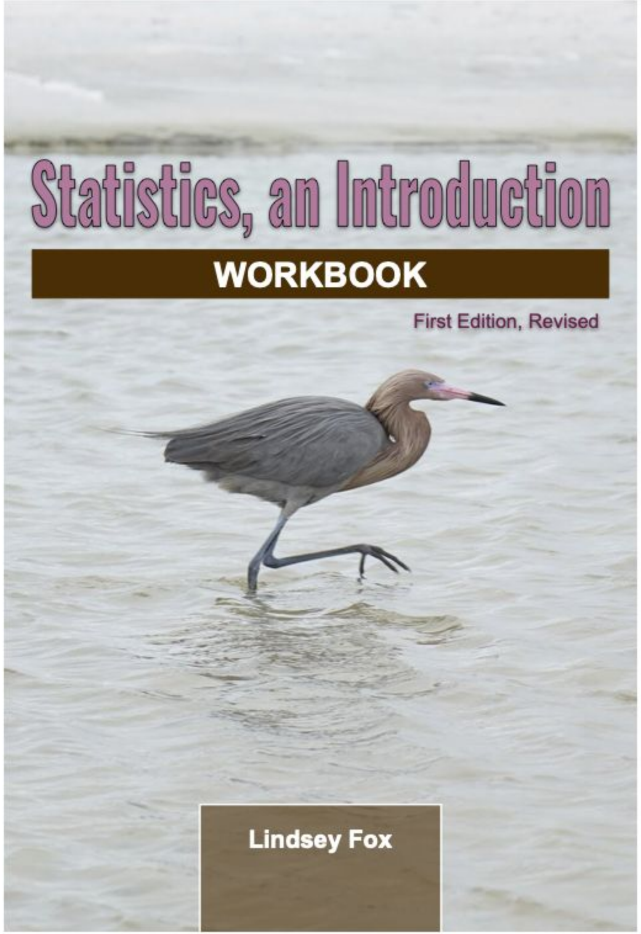
- Introduce the course in which I use my activity
- Context for activity in the curriculum
- Activity: Transformations of Random Variables

Statistics, an Introduction (MA 133)

- Required by many majors on campus
- First math course in years for many students
- Math-phobic students can become disconnected from the material when using computational tools
- Main computational tool: TI 83/84 calculator
- Other tools:
  - Random.org
  - Google Sheets
  - StatCrunch (Pearson)

Statistics, an Introduction (MA 133)

- My workbook: note-taking guides and worksheets
- Many worksheets/activities are “tactile” and require students to explore course material with pencil and paper



Curriculum

- Samples
- Elementary Probability
- Discrete Populations
- Exam #1
- Continuous Populations
- Sampling Distributions
- Exam #2
- Hypothesis Testing
- Confidence Intervals
- Exam #3
- Linear Correlation
- Introduction to Linear Regression
- Final Exam/Final Project

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Transformations of Random Variables

Activity: Transformations of Random Variables

$\frac{X-4.5}{2.247}$

X	$P(X)$	$X-3$	$P(X-3)$	$X+10$	$P(X+10)$	$2X$	$P(2X)$	$X/10$	$P(X/10)$	$\frac{X-\mu_X}{\sigma_X}$	$P(\frac{X-\mu_X}{\sigma_X})$
0	0.04	-3	0.04	10	0.04	0	0.04	0	0.04	-2.003	0.04
1	0.06	-2	0.06	11	0.06	2	0.06	0.1	0.06	-1.558	0.06
2	0.10	-1	0.10	12	0.10	4	0.10	0.2	0.10	-1.113	0.10
3	0.14	0	0.14	13	0.14	6	0.14	0.3	0.14	-0.668	0.14
4	0.16	1	0.16	14	0.16	8	0.16	0.4	0.16	-0.223	0.16
5	0.16	2	0.16	15	0.16	10	0.16	0.5	0.16	0.223	0.16
6	0.14	3	0.14	16	0.14	12	0.14	0.6	0.14	0.668	0.14
7	0.10	4	0.10	17	0.10	14	0.10	0.7	0.10	1.113	0.10
8	0.06	5	0.06	18	0.06	16	0.06	0.8	0.06	1.558	0.06
9	0.04	6	0.04	19	0.04	18	0.04	0.9	0.04	2.003	0.04

$\mu_X = 4.5$ $\mu_{X-3} = 1.5$ $\mu_{X+10} = 14.5$ $\mu_{2X} = 9$ $\mu_{X/10} = 0.45$ $\mu_{\frac{X-\mu_X}{\sigma_X}} = 0$

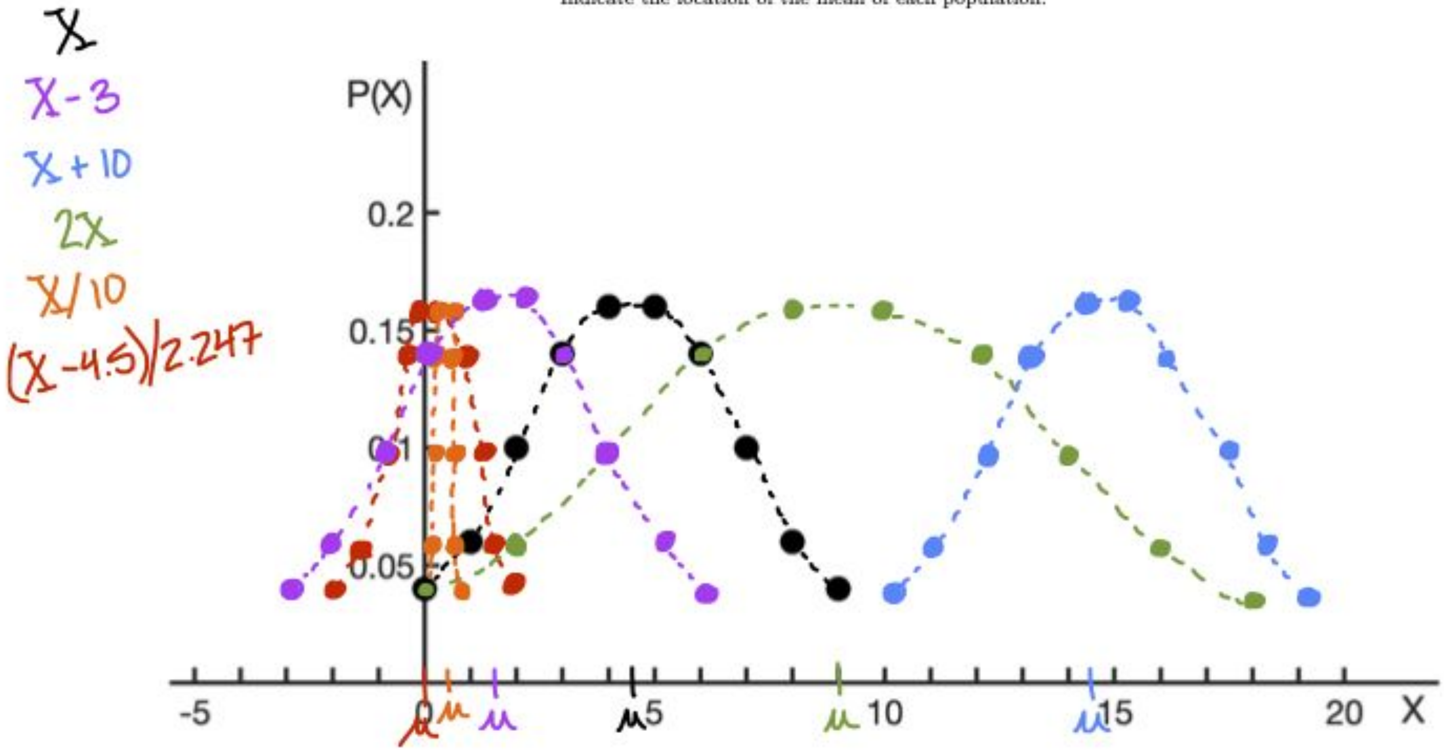
$\sigma_X = 2.247$ $\sigma_{X-3} = 2.247$ $\sigma_{X+10} = 2.247$ $\sigma_{2X} = 4.494$ $\sigma_{X/10} = 0.2247$ $\sigma_{\frac{X-\mu_X}{\sigma_X}} = 1$

What patterns do you notice?

The operation applied to X is also applied to μ.

Multiplication + divison only apply to σ

Use different colors or dot shapes (stars, squares, triangles, etc.) to differentiate between the populations.
Indicate the location of the mean of each population.



Population X is symmetric. What shape best describes the transformed populations? Symmetric

Desmos Link

2. Population Y is a skewed right, ***continuous*** population with the following mean and standard deviation:

$$\mu_Y = 4 \qquad \sigma_Y = \sqrt{8}$$

Based on what you have observed with population X, make an educated guess at what the parameters for the transformed populations of Y will be.

$Y - 3$: $\mu_{Y-3} =$ $4-3=1$ $\sigma_{Y-3} =$ $\sqrt{8}$

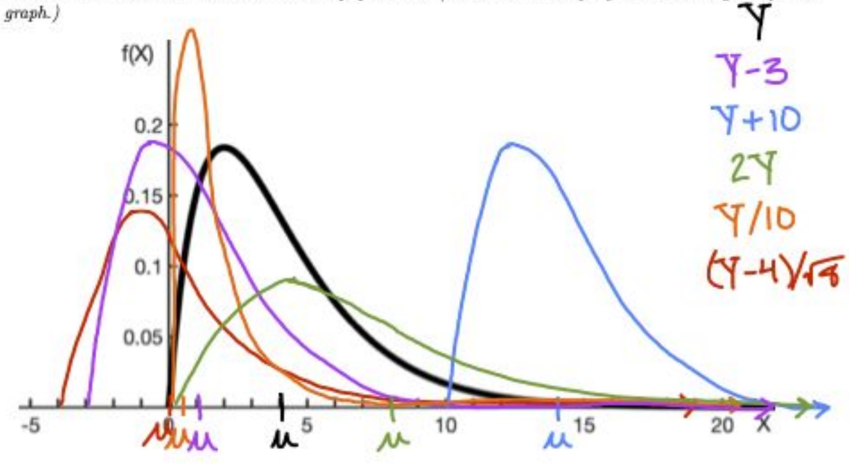
$Y + 10$: $\mu_{Y+10} =$ $4+10=14$ $\sigma_{Y+10} =$ $\sqrt{8}$

$2Y$: $\mu_{2Y} =$ $2\cdot 4 = 8$ $\sigma_{2Y} =$ $2\sqrt{8}$

$Y/10$: $\mu_{Y/10} =$ $4/10 = 0.4$ $\sigma_{Y/10} =$ $\sqrt{8}/10$

$\frac{Y-\mu_Y}{\sigma_Y}$: $\mu_{\frac{Y-\mu_Y}{\sigma_Y}} =$ $\frac{4-4}{\sqrt{8}} = 0$ $\sigma_{\frac{Y-\mu_Y}{\sigma_Y}} =$ $\sqrt{8}/\sqrt{8} = 1$

The plot of population Y is given below. Draw the plots for the transformed populations on the same graph. Use different colors or lines (solid, dotted, starred, etc.) to differentiate between the populations. Indicate the location of the mean of each population. *(Hint: Think carefully about the heights of each graph.)*



What shape best describes the transformed populations? Skewed right

Results: DO NOT complete this page on your own! We will work together as a class.

	Population	Mean	Standard Deviation	Variance	Distribution Shape	
	X	μ	σ	σ^2	uniform, symmetric, skewed, etc.	
shift	$X - c$	$\mu - c$	σ	σ^2	same	← only μ affected
stretch/shrink	cX	$c\mu$	$c\sigma$	$(c\sigma)^2 = c^2\sigma^2$	same	← μ & σ affected
center	$X - \mu$	$\mu - \mu = 0$	σ	σ^2	same	
standardize	$\frac{X - \mu}{\sigma}$	$\frac{\mu - \mu}{\sigma} = 0$	$\frac{\sigma}{\sigma} = 1$	$1^2 = 1$	same	

Examples: Consider the normal population X with $\mu = 100$ and $\sigma = 5$. Find the following parameters.

1. $\mu_{X-9} = 100 - 9 = 91$

   $\sigma_{X-9} = 5$

2. $\mu_{3X} = 3 \cdot 100 = 300$

   $\sigma_{3X} = 3 \cdot 5 = 15$

3. $\mu_{\frac{X-100}{5}} = \frac{100-100}{5} = 0$

   $\sigma_{\frac{X-100}{5}} = \frac{5}{5} = 1$

4. On an exam, the mean score was 82.6 with a standard deviation of 4.75. If the instructor curves all scores by 5 points, what will be the curved mean and standard deviation? (This is a discrete population!)

   $\mu_X = 82.6$ $\sigma_X = 4.75$ $\mu_{X+5} = 87.6$ $\sigma_{X+5} = 4.75$

5. At the beginning of spring, sunflowers average 10.4 inches in height with a standard deviation of 2.33 inches. By the end of summer, sunflowers quadruple in height. What is the mean and standard deviation of the height of sunflowers at the end of summer? (This is a continuous population!)

   $\mu_X = 10.4$ $\sigma_X = 2.33$ $\mu_{4X} = 41.6$ $\sigma_{4X} = 9.32$

Next few activities and notes

- Worksheet: Introduction to Sampling Distributions
  - Explore all possible samples of a given size from a small population and their statistics
  - **Understand the mean and standard deviation of sampling distributions, and connect them back to the random variable sampled from**
- Worksheet: The Distribution of Sample Proportions with Dice
  - Explore the shape of the proportion sample distribution as the sample size increases
  - Generate dice data on Random.org
  - Explore central limit theorem on StatCrunch
  - **Understand the shape of sampling distributions**

Next few activities and notes

- Notes: Sampling Distributions
  - How to calculate $P(\bar{X} > \#)$, for example
  - Use transformations to change sample mean sampling distribution into the standard normal distribution, for example
  - **Turn this new probability question into something we already know how to do**

Takeaway

- A simple activity in between the two major topics for the next exam
- Makes the transition from the “easier” material of the first exam to the “harder” material of the rest of the course smoother
- Gives the students an active down day during a busy part of the semester

Statistics, an Introduction: Workbook

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Kendall Hunt Publishing

Thank you! Questions?